Newtonian Cosmology

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• **From Hubble's 1929 discovery, we know that our universe is expanding**

• **The observed distribution of matter and radiation in the universe display homogeneity and isotropy on scales greater than $\sim$ few hundreds of mega-parsecs (Mpc)**

1 pc = 3×10^{16} m ; 1 kpc = 10^3 pc ; 1 Mpc= 10^6 pc

• **Physical (proper) distance between two points separated by a radial coordinate interval r is $\cong a(t)r$**

• **$a(t)$ is the expansion scale factor and $\dot{a} \equiv da/dt$ is the rate of expansion if $\dot{a} > 0$**

• **If a galaxy G does not have any peculiar velocity, then its radial coordinate r_G (with our Milky Way galaxy as the origin) does not change with time**

• **However, its proper distance (or, the physical distance):**

$$l_G(t) = a(t)\ r_G$$

evolves with time

• **Comoving radius (or, comoving distance): This is the radial coordinate r of a spatial point so that the proper distance $l(t)$ of it from us is simply is $a(t)$ times the comoving distance:**

$$l(t) = a(t)\ r$$

• **The rate at which a spatial point with a fixed comoving radius is moving away from us is given by:**

$$v(t) = \frac{dl(t)}{dt} = \frac{d[a(t)r]}{dt} = r\ \frac{da(t)}{dt} = r\ \dot{a} = \frac{\dot{a}}{a}\ a(t)\ r = \frac{\dot{a}}{a}\ l(t)$$

• **What does this remind you?**

Of course, the Hubble's law !

$$v = H_0\ l$$

• **Hubble parameter:**

$$H(t) \equiv \frac{\dot{a}}{a}$$

$$H_0 \equiv \frac{\dot{a}}{a}\Big|_{t=t_0}$$

• **A big tension concerning the value of the present Hubble parameter:**

CMB precision data $\Rightarrow H_0 \cong 67.4$ **km/s/Mpc, while accurate measurements of distances of nearby galaxies** $\Rightarrow H_0 \cong 74$ **km/s/Mpc**

• **How does one find the scale factor** $a(t)$**?**

• **Assuming that matter and radiation are homogeneously and isotropically distributed, Friedmann-Le Maitre-Robertson-Walker (FLRW) Cosmological Models use general relativity to set up dynamical equations for** $a(t)$ **and then obtain solutions**

• **In these lectures, we will use Newtonian models to arrive at similar equations**

Consider a very large sphere of comoving radius r at a cosmic time t

At time t, the proper radius of the sphere is $l(t) = a(t)\ r$ and the proper speed of the surface is $v(t) = H(t)l(t)$

The density anywhere inside the sphere at time t is $\rho(t)$

Hence, mass of the sphere is:

$$M_r = \frac{4\pi}{3} l^3(t)\rho(t)$$

The energy of a test particle of mass m on the surface of this sphere is:

$$E = \frac{1}{2}mv^2(t) - \frac{GM_r m}{l(t)} = m\Big[\frac{1}{2}\ H^2\ l^2(t) - \frac{G}{l(t)}\ \frac{4\pi}{3} l^3(t)\rho(t)\Big] \ \Rightarrow$$

$$\frac{E}{m} = \frac{l^2(t)}{2}\Big[H^2(t) - \frac{8\pi G\rho(t)}{3}\Big] = \frac{r^2 a^2(t)}{2}\ \Big[\ \frac{\dot{a}^2}{a^2} - \frac{8\pi G\rho(t)}{3}\ \Big] \ \Rightarrow$$

$$\frac{\dot{a}^2}{a^2} - \frac{8\pi G\rho(t)}{3} = \frac{2E}{mr^2 a^2}$$

- **Since E, r and m are constants, we may write the above equation as:**

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho(t)}{3} \quad \textbf{(First FLRW equation)}$$

where,

$$k \equiv -\ \frac{2E}{m\ c^2\ r^2}$$

- **So,** $k = 0$ **corresponds to a model when the Newtonian energy of the sphere of comoving radius** r **is zero i.e. the sphere will just be reaching an infinite physical size as** $t \to \infty$

  $\Rightarrow$ **Spatially Flat FLRW model**

- $k = -1$ **corresponds to a model when the Newtonian energy of the sphere of comoving radius** r **is positive i.e. the sphere will reach an infinite physical size as** $t \to \infty$

  $\Rightarrow$ **Open FLRW model**

- $k = +1$ **corresponds to a model when the Newtonian energy of the sphere of comoving radius** r **is negative i.e. the sphere will contract after reaching a maximum physical size**

  $\Rightarrow$ **Closed FLRW model**

• **Critical density:**

What is the condition for the universe to be described by a k=0 model?

Now, from the first FLRW equation, if $k = 0$ **then:**

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho(t)}{3} \Rightarrow H^2 = \frac{8\pi G\rho(t)}{3}$$

so that, at the current epoch,

$$H_0^2 = \frac{8\pi G\rho(t_0)}{3} \Rightarrow \rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

If we write the present day Hubble parameter in the form:

$$H_0 = 100\ h_0\ \textbf{km/sec/Mpc}$$

then the critical density at the present epoch:

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29}\ h_0^2\ \ \textbf{gm/cm}^3$$

• **If the present density** $\rho_0 > \rho_{c,0}$ **then the universe is described by a** $k = +1$ **model**

• **If the present density** $\rho_0 < \rho_{c,0}$ **then the universe is described by a** $k = -1$ **model**

• **Causally connected regions have size less than the Hubble radius,** $R_H(t) = c/H(t) \sim c\ t$**.**

FLRW dynamics:

$$\frac{\dot{a}^2}{a^2} \cong \frac{8\pi G}{3}\rho \quad \textbf{and} \quad \frac{d(c^2 \rho a^3)}{da} + 3pa^2 = 0$$

• **Early universe:** $a(t) \propto \sqrt{t} \quad \Rightarrow \dot{a} \propto 1/\sqrt{t} \quad \Rightarrow$

$$\frac{\dot{a}}{a} \sim \frac{1}{t} \quad \Rightarrow \quad \rho(t) \sim \frac{3}{8\pi G\ t^2}$$

• **Mass within the Hubble radius:**

$$M_H(t) \sim \frac{4\pi}{3}(R_H(t))^3 \rho(t) \sim \frac{4\pi}{3}(c\ t)^3 \rho(t) = \frac{c^3 t}{2G} = 2{\times}10^3 \left(\frac{t}{10^{-35}\ s}\right) \textbf{gm}$$

• **Gravitational Radius associated with** $M_H(t)$**:**

$$\frac{2GM_H(t)}{c^2} \sim \frac{2G}{c^2}\left(\frac{c^3 t}{2G}\right) = c\ t \sim R_H(t)$$

$\Rightarrow$ **Any excess mass in the Hubble radius due to fluctuations** $\rightarrow$ **Gravitational collapse** $\rightarrow$ **Formation of Primordial Black holes (PBHs)**

(Hawking, 1971; Carr and Hawking, 1974; Carr, 1976;....)